defining the decibel

Why bother? Because in electronics — as in any science — definitions *do* make a difference.

The unit of the decibel, or dB, is used quite widely in electronics to express everything from amplifier gains to bandwidth ratios (is that 10 log or 20 log for bandwidths?). Often the question, "Is that dB-voltage or dB-power?" is heard. The answer to that question is an unequivocal "yes." The short article that follows reiterates the apparently long-forgotten history of the decibel and discusses both its proper application and a few of its common misapplications as well.

The decibel is, roughly, the smallest change in acoustic power that the ear can detect.[1] It's one tenth of a bel, a unit named for Alexander Graham Bell, whose original research revealed the logarithmic amplitude response of the human ear;[2] not surprisingly, the concept of the bel was originally used in the field of telephony. But the unit was found to be too large for practical application, and the decibel was soon found to be more convenient.

In the original acoustic terms, the decibel was defined as 10 log to the base of 10 of the ratio of two acoustic intensities (powers). (A similar but much less frequently used unit is the *Neper* — from Napier — which is given as 1 log to the base e of a *voltage* ratio.[2] Yes, the multiplier is 1, not 10.) In modern electronics, however, the decibel is *defined* as 10 log of a power ratio in which the two powers ratioed are measured at a particular point in a system — at the output of an amplifier, for example. *This is the only definition. Other descriptions of the decibel, such as 20 log of a voltage ratio, are derivations of this definition, often with some critical information omitted.*

The decibel is really just a type of mathematical shorthand. It is more convenient, for example, to express the power gain of an amplifier as 80 dB than as 100,000,000 watts/watt. One variation to this basic definition has been a generalization to allow the two powers ratioed to be at *different* points in a system that have equal impedances — for example, the power gain of an amplifier in a constant 50-ohm system expressed in dB as 10 log of the ratio of the output power to the input power. Such a generalization, however, is still consistent with the original definition. Consider a 50-ohm amplifier in a 50-ohm system. Its input and output impedances must both be 50 ohms to be consistent with the 50-ohm system. Therefore, the source driving the amplifier will deliver the same power to a 50-ohm termination as it delivers to the amplifier input. Let us choose the 50-ohm input to a power meter as the point of measurement of the original definition above. First apply the source directly to the power meter input and record the source power. Then, remove the source from the measurement node (power meter input), apply it to the amplifier input, and apply the amplifier output to the power meter. Measure the new power at the point of measurement. The amplifier gain in dB is then 10 log of the ratio of the second measurement, the output power, to the first measurement, the power applied to the input. This measurement technique is a direct application of the definition of the decibel.

Alternately, we could, by some means, measure the input and output powers of the amplifier with it attached to the source and 50-ohm load (computed from measured input and output potentials perhaps) and compute the gain in a similar manner as above. There is a subtle difference between this second measurement technique and the first. In the first, a single point of measurement, the input to the power meter, was used to measure the two powers for the ratio; in the second, two different points in the system were observed — the input and output ports of the amplifier.

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Since the system was defined to be a 50-ohm system throughout, both techniques will yield the same results. However, if the impedances in the system are not the same throughout, the results will *not* be the same. (This will be demonstrated later.) So, the ratio of two powers, P_2 and P_1, in a constant impedance system expressed in dB, is given by **eq. 1**, where the term "*D*" is simply a general notation and the form "*dB*" is used to show the units of the result.

$$D = 10 \log (P_2/P_1)\ [dB] \qquad (1)$$

In many cases the power, P_1, is chosen as some convenient reference power such as one milliwatt. The value of D is then given in dB referred to a milliwatt, abbreviated dBm. This is still consistent with the basic definition of the dB since D is then a representation of the actual power at a point in a system compared to the chosen reference power at that same point.

Now, we will expand **eq. 1**. However, since this is not a lesson in arithmetic, the impedances will be defined as being real with no imaginary part, which will simplify the math considerably. Each of the powers in **eq. 1** may be expressed in terms of the potentials and corresponding impedances, or resistances for the case of impedances with only a real component, at the power measurement points. Expanding **eq. 1**:

$$D = 10 \log [(E_2^2/R_2)/(E_1^2/R_1)] \qquad (2)$$

$$= 10 \log (E_2^2/E_1^2) - 10 \log (R_2/R_1) \qquad (3)$$

$$= 20 \log (E_2/E_1) - 10 \log (R_2/R_1)\ [dB] \qquad (4)$$

This is an interesting result. We can now see where the 20 log of a voltage ratio expression originated in the widely used dB expressions. But what about the second term in **eq. 4**? Well, in the case of a constant impedance system, the two resistances are the same value, resulting in a second term of 10 log(1), which is of course zero. As a result, D is correctly expressed in dB as 20 log of the ratio of two voltage measurements, which is the expression so familiar to many of us. So we'll make a note of it here:

FOR CONSTANT IMPEDANCE ONLY!!!

$$D = 20 \log (E_2/E_1)\ [dB] \qquad (5)$$

This would be a good point to digress a moment and examine the question, "Is it dB-voltage or dB-power?" Consider a 50-ohm amplifier in a constant 50-ohm system. If we applied an input signal of one microwatt (−30 dBm right?) to this amplifier and measured an output power of one milliwatt (0 dBm?), what would be the gain of that device in dB? Letting that power ratio be represented by "G" and applying the basic definition of the dB given in **eq. 1**:

$$G = 10 \log (P_o/P_{in})\ [dB]$$

$$G = 10 \log (1\ \text{milliwatt}/1\ \text{microwatt}) \qquad (6)$$

$$= 10 \log (1000) = 30\ dB$$

We could have easily found this result by *subtracting* the −30 dBm input level from the 0 dBm output — 0 dBm − (−30 dBm) = +30 dB. To show this mathematically, consider the following:

$$G\ [dB] = 10 \log (P_o/P_{in})$$

$$= 10 \log (P_o/P_{in}) + 10 \log (1\ mW/1\ mW)$$

$$[\text{add } 10 \log (1\ mW/1\ mW) \text{ which } = 0]$$

$$= 10 \log (P_o/1\ mW) - 10 \log (P_{in}/1\ mW)$$

$$= P_o\ [dBm] - P_{in}\ [dBm]$$

Now we will try it from a voltage point of view. First we must compute what input and output voltages we would measure with the corresponding powers given. We all know that the power dissipated in a resistor with a potential E applied across it is given by:

$$P_R = E^2/R\ [watts]$$

Solving for E in terms of P and R:

$$E = (P \cdot R)^{1/2}\ [Volts,\ RMS]$$

For the one microwatt input (remember, we said we had a 50-ohm system):

$$E\ (1 \mu W) = (1 \cdot 10^{-6}\ watts \cdot 50\ ohms)^{1/2}$$
$$= 7.07\ millivolts\ RMS$$

And for the one milliwatt output:

$$E\ (1 mW) = (1 \cdot 10^{-3}\ watts \cdot 50\ millivolts)^{1/2}$$

$$= 223.6\ millivolts\ RMS$$

Now, applying **eq. 4**:

$$G = 20 \log (E_2/E_1) - 10 \log (R_2/R_1)\ [dB] \qquad (7)$$

$$G = 20 \log (223.6\ mV/7.07\ mV) - 10 \log (50\ ohms/50\ ohms)$$

$$= 20 \log (31.63) - 0 = +30\ dB$$

Look carefully at the two results in **eqs. 6** and **7**. If you were told that the gain of this 50-ohm amplifier was 30 dB, would you have to ask "dB-voltage or dB-power?" (I hope not.) As shown in this example, computing the gain by either 10 log of the power ratio or 20 log of the voltage ratio yields exactly the same result in a constant impedance system. So you can see that the all-too-often-asked question has little meaning when the concept of the decibel is properly used.

We will now see how the concept came to be misapplied. Look back at **eq. 4**. This is an exact expression of the decibel and, as explained, reduces to **eq.**

5 in systems of constant impedance. In the early applications of the decibel, power measurements were made in waveguide and coaxial RF systems using various instruments for direct power measurement. The use of these instruments required the signal of interest to be applied directly to the measurement instrument input as is required by the definition of the decibel. Therefore, ratios of measured powers expressed in dB were consistent with the original definition. Then something terrible happened: the performance of electronic instruments improved dramatically. RF voltmeters could now be used to measure actual RMS potentials in systems, rather than only power. Oscilloscopes could provide direct viewing of the voltage waveforms from which RMS values could be computed. And worse yet, these instruments were of such a nature that these potential measurements could be made quite accurately in systems of almost any impedance without the need to break the circuit for direct application of the measured signal to the measuring instrument. In fact, the impedance did not even have to be known to accurately measure potentials, although circuit loading did have to be considered. Well, many of the first applications of these instruments were still in the area of constant impedance systems and it was well known that **eq. 5** applied, and **why**. As several generations of engineers and technicians used these new and ever-improving instruments, the use of **eq. 5** became second nature and its origin (and limitations) slowly became lost and forgotten. Then another terrible thing happened . . . the operational amplifier appeared. These were marvelous devices with staggeringly high voltage gains — perhaps as high as 1,000,000 or even higher! Using such large terms in everyday communication presented a bit of an inconvenience. Then someone, remembering **eq. 5** (at least most of it), said "Wow, we can express this gain in dB as 20 log of the voltage gain." What was omitted was that **eq. 5** applies only in constant impedance systems. Operational amplifiers typically exhibit very high input and very low output impedances. So was yet another misapplication of the decibel born.

To demonstrate the problem associated with this misapplication of the concept, we will examine a few examples. Consider an operational amplifier configured for a voltage gain of unity. Let the amplifier have a 1 Megohm input resistance and a very low output resistance (much smaller than 50 ohms). Also, consider a source with a 50-ohm impedance. Finally, let the load be 50 ohms. If we apply an input signal and measure the input and output voltages we will naturally find them to be the same since the amplifier is configured for a gain of one. Using **eq. 5** and rather ignoring the impedance requirement, we would find the amplifier gain to be 0 dB. Now let's compute the gain in dB as 10 log of the output to input power ratio. The input power is simply the input RMS voltage squared, divided by the input resistance. The output power is given as the output RMS voltage squared divided by the load resistance. However, since the voltage gain is unity, the input and output voltages are equal. Let that voltage be E. Computing the gain from the power ratio:

$$G = 10 \log [(E^2/R_L)/(E^2/R_{in})]$$
$$= 10 \log (R_{in}/R_L)$$
$$= 10 \log (1 \cdot 10^6 \text{ ohms}/50 \text{ ohms})$$
$$= 10 \log (2 \cdot 10^4) = 43 \text{ dB}$$

Well, that presents a bit of a problem. Is the actual gain 0 dB or 43 dB? Let's try still a different measurement by trying to apply the single-point measurement approach of the original definition. Let the 50-ohm load be the input resistance of a 50-ohm power meter. Applying the source to the power meter input, a power, P, is observed. Now, move the power meter to the amplifier output and apply the source to the amplifier input. Since the source is not loaded by the amplifier input (1 megohm $>>$ 50 ohms), the voltage at the amplifier input is twice that measured when the source was terminated with the 50 ohms of the power meter. (This can easily be shown with some simple circuit analysis, but since that's not our purpose here, you'll have to either accept it as true, or prove it for yourself.) The amplifier output voltage is also twice the loaded value of the source, since the amplifier voltage gain is unity and the low output resistance of the amplifier prevents loading by the power meter. Power varies as the square of voltage, so the doubling of the voltage at the power meter input results in an increase in power by a factor of 4. The power reading of the power meter will then be 4P. Applying **eq. 1**:

$$G = 10 \log (P_o/P_{in})$$
$$= 10 \log (4P/P) = 6 \text{ dB}$$

This gives us still another choice as to what the gain in dB is for an operational amplifier configured for unity gain. It is either 0 dB, 43 dB, or 6 dB, depending how one makes the measurement.

Now, let's modify the unity gain amplifier circuit configuration slightly by the addition of an input transformer. Let that input transformer match the 1 megohm amplifier input resistance to the 50-ohm source resistance, a turns ratio of 1:141 (remember, transformer impedances vary as the square of the turns ratios). The transformer/amplifier combination now satisfies the constant impedance requirement of the definition of the decibel — the input and output resistance is 50 ohms in a 50-ohm system. The 50-ohm input of the transformer presents the same load as the power meter of the original measurement of the source power. The source power will produce some input voltage. Let that voltage be E. Since the transformer has a 1:141 turns ratio, the input potential to the amplifier is 141E and since the amplifier has unity gain, the output potential is 141E. Since this is a constant impedance system, we may compute the power gain either from the power ratio or the voltage ratio. Therefore, since we have the input and output voltages (in terms of E) we will apply **eq. 5**.

$$G = 20 \log (E_2/E_1)$$
$$= 20 \log (141E/E) = 43 \text{ dB}$$

Now, isn't that an interesting result? This is the same value that was found for the original configuration in the second calculation, which was based on the actual input and output powers. This can somewhat be understood since the transformer can have no power gain. The transformer does provide a proper impedance match to the source so that for any given source potential the maximum amount of power will be coupled, but it does not provide any power gain. The optimum matching simply implies that this is the configuration in which the *available* source power is most effectively coupled. Thus, for some desired output power, this configuration will require the least input voltage, and least available input power. The first unity gain configuration and the transformer/amplifier configuration both have a power gain of 20,000. However, in the first, the voltage gain is unity, so if an output potential of V volts RMS is needed for some desired output power, the input potential must also be V volts. In the transformer/amplifier configuration, the voltage gain is 141, so for an output of V volts an input of V/141 volts is needed. Since each of these configurations is delivering the same power to the load and they have the same power gain, the second makes the more efficient use of the source signal.

Since the gain of this final configuration to which the concept of the decibel may be properly applied is the same as that of the simple unity gain amplifier expressed in dB as 10 log of the actual output to input power ratio, perhaps the gain in dB of the unity voltage gain amplifier should be 43 dB? One can easily see the confusion that can result. This is particularly true since all four of these results could be considered correct depending upon how one chooses to use the concept of the decibel. Considering the first two cases it might have some meaning to use the expressions dB-voltage and dB-power to distinguish between the two measurement techniques. But what do we do with the third measurement? It was also a power measurement and actually more consistent with the actual definition of the dB. Perhaps this value could be called "dB-power-almost-consistent-with-the-definition-of-the-decibel," or dB-PACWTDOTD for short. Certainly that would be a bit ridiculous — but would it be any more absurd than dB-voltage and dB-power? Then there's the modified configuration, to which the concept of the decibel may be properly applied, which yields a power gain that is the same as that of the second calculation. However, this is a different configuration, and its relation to the problem is not immediately obvious. Perhaps the best way to avoid this dilemma is to stick to the actual definition of the decibel. Nevertheless, widespread use of the expression of voltage gain in dB as 20 log of the ratio of the output to input voltages has resulted in this expression becoming a type of alternate definition of the decibel.

One popular use of this expression is the specification of the open loop voltage gain of operational amplifiers. There seems to be a slight trend away from this erroneous specification, however, in favor of the correct units of volts per millivolt, volts per microvolt, etc. Look through some data books with specifications on operational amplifiers and see if you can find these different open loop gain units.

noise analysis — yet another creative misapplication

The use of the decibel to express the voltage gain of an amplifier in a system of non-constant impedances is by far the most common misapplication of the decibel that you're likely to encounter. Yet there's another interesting misapplication that's also quite creative; this is found in the area of noise analysis. The noise power available in a system is directly a function of the system noise bandwidth. Without going into detail, the available noise power, P_n, in a noise bandwidth, BW_n, and absolute temperature, T_O, is given by **eq. 8**.

$$P_n = k \cdot T_O \cdot BW_n \text{ (watts)} \quad (8)$$
$$k = \text{Boltzmann's constant}$$
$$= 1.38 \cdot 10^{-23} \text{ Joules/degrees Kelvin}$$

Now suppose we have two different noise bandwidths and want to know how much noisier, in dB, the larger is than the smaller. Consider an amplifier system of power gain G with a variable bandpass filter and a suitable power meter tied to the amplifier output. With the bandpass filter set for a narrow bandpass, BW_{n1}, let the power reading be P_{n1}. Then let the filter be adjusted to a wider bandpass, BW_{n2}, with a corresponding power reading of P_{n2}. The relative increase in power with the increased bandwidth expressed in dB is then given as $10 \log (P_{n2}/P_{n1})$, and this expression is an exact application of the definition of the dB. Let this quantity be defined as D. The two powers may be expressed in the form of **eq. 8** above. Then, expanding the expression for D:

$$D = 10 \log (P_{n2}/P_{n1})$$
$$= 10 \log (k \cdot T_o \cdot BW_{n2} \cdot G / k \cdot T_o \cdot BW_{n1} \cdot G) \quad (9)$$
$$= 10 \log (BW_{n2}/BW_{n1}) \text{ (dB)}$$

Well, there you have it — the ratio of two bandwidths expressed in dB is given as 10 log of the bandwidth ratios. Just what does it mean? That's right, *nothing* . . . unless, of course, you know that you're really talking about ratios of noise powers and not simply bandwidths. Feel free to consider some of the possibilities for this misapplication — but please don't take them seriously. Why, one could use **eq. 9** (without considering its origin) to express the peaking properties of a bandpass filter in dB as 10 log of the output bandwidth to input bandwidth! Of course in this case we'd most likely want to invert the ratio (i.e., 10 log of the input to output bandwidths) so that we would always have positive values to work with. We could even express quality factor, Q, in dB if we are a little creative. Quality factor of various systems is often expressed as the ratio of the center frequency to the bandwidth. Well, the center frequency could be considered as a bandwidth with the lower frequency of 0 Hz. Then Q would be nothing more than a ratio of two bandwidths and misusing **eq. 9**, we could express Q in dB (again, we would wish to invert the ratio to have positive dB values of Q since negative values might be confusing).

If these suggestions seem quite outrageous, perhaps it's because they haven't been seen before. These are actually as correct as expressing the open loop voltage gain of an operational amplifier in dB, but we've come to know *that* expression because of its widespread use, and so as a result, it doesn't look particularly strange. But this doesn't make it inevitably correct. There is some obvious confusion because in the absence of uncertainty, the voltage/power question would need never be asked. If you stick to using the concept of the decibel only where it applies, you'll have no problem. In cases where you must work from someone else's error, you'll just have to try to figure out what was meant. This does not mean that we would specify the gain of the unity gain amplifier in the example above as 43 dB. That voltage gain is 1 volt/volt or simply a voltage gain of 1. The power gain of that configuration is 20,000. That is where we arrived at the 43 dB figure; but as pointed out, the concept of the decibel does not apply because the system is not of constant impedance. The power gain would simply be stated as 20,000 watts/watt, 20 watts/milliwatt, or simply a power gain of 20,000.

As the examples above have shown, misapplication of the concept of the decibel has led to considerable confusion as to what is actually implied by the expression of a quantity in dB. Hopefully this article has served to clarify your understanding of the concept, and you can now correctly apply the concept. Furthermore, when you see quantities expressed in dB, you'll be able to tell whether or not they're proper expressions. In cases where they're not, you should have a better understanding of the concept to aid you in trying to comprehend the original intention. In any event, if you keep the definition of the decibel clearly in mind and always apply it properly, then your expressions, at least should always be correct and should be understood by anyone who shares your understanding of the decibel. When you're asked, "Is that dB-voltage or dB-power?", you'll not only be able to show why that question has little meaning, but also silently revel in your mastery of the concept. Always return to the basic definition of a notation or concept and apply it accordingly. Also, look those basic definitions up in the literature. Don't simply take some *expert's* opinion as being correct. Perhaps you should apply that advice to the information in this article!

references

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