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OPTIMIZING AMPLIFIER GAIN-BANDWIDTH PRODUCT

An analysis of multistage-amplifier behavior

It often seems that no matter what the gain or bandwidth of an amplifier, I find myself needing a little more of one or the other, or both, in my applications. Unfortunately, when I try to add another amplifier stage to improve one of these parameters, the other suffers. For example, I may add a second amplifier stage to increase gain only to find that now I don't have enough bandwidth. With multistage amplifiers, there's an optimum gain for each stage that will maximize the overall gain-bandwidth product of the total amplifier system. But it's quite possible that you'll find this optimum-stage gain a bit surprising.

In the discussion which follows, I show the computations for optimum stage gain. Although this result is relatively well known and the derivation straightforward, it's somewhat difficult to find references describing the work in any detail.[1] In any event, I hope you'll find the information here of interest and of some value in your amplifier design work.

Gain versus bandwidth

Many amplifiers, like operational amplifiers and some RF amplifiers, have a constant gain-bandwidth product. This means that if you increase the gain, you have less bandwidth, and vice versa.

An operational amplifier has this characteristic. The industry-standard 741 operational amplifier has a nominal bandwidth of 10 Hz and a nominal voltage gain of about 100,000—a gain-bandwidth product of 1 MHz. If you lower the gain to 1000 by using a feedback network, the bandwidth will increase to 1 kHz. Suppose that you were to use two 741 amplifiers in a two-stage amplifier circuit with each stage having a gain of 316. The bandwidth of each amplifier would be about 3.2 kHz.

The total gain of the pair of amplifiers would be 100,000 and the combined bandwidth about 2.1 kHz. The gain-bandwidth product will have been increased to 210 MHz—a factor of more than 200 over that of a single 741. Suppose four amplifiers, each with a gain of 17.78 and a bandwidth of 56.2 kHz, are used. The total gain will still be 100,000, but the bandwidth will be 24.4 kHz. The gain-bandwidth product is 2.44 GHz—an increase of more than a factor of 2000 over that of a single amplifier.

You can continue this process, but you'll reach a point where a further reduction in amplifier gain and an increase in the number of stages will decrease the gain-bandwidth product. For example, suppose that the new amplifier gain selected is unity. The bandwidth of a single stage would be 1 MHz. No matter how many of these amplifier stages you use, the gain will remain at unity; however, the bandwidth will be reduced as you add more stages. So the best gain-bandwidth product you can obtain with a stage gain of unity is 1 MHz, using a single stage. That's considerably lower than the results obtained with four stages. Because a reduction in gain per stage from 100,000 with multiple stages results in an increase in gain-bandwidth product, but using a gain of unity results in a lower gain-bandwidth than that of the four-stage case, you'd expect that there's an optimum gain that will provide the best possible gain-bandwidth product.

Amplifier bandwidth and midband gain

Assume an amplifier with a midband voltage gain g_0 and a lower frequency response to DC, or near DC. Let the upper cutoff frequency, the frequency where the gain is 3 dB below the midband value, be f_c. The amplifier bandwidth is defined as the bandwidth between the lower and upper cutoff frequencies. Because the lower cutoff frequency is far below the upper frequency, the amplifier bandwidth (BW) is essentially equal to the upper cutoff frequency, BW $\approx f_c$. Also, let this amplifier have a constant gain-bandwidth product $g_0 \times f_c$ that is a constant k for any gain. The amplifier voltage gain as a function of frequency is given by:

$$A_v(f) = \frac{g_0}{[1 + j(f/f_c)]} \quad (1)$$

The term g_0 is the midband gain of the amplifier and f_c is the upper cutoff frequency. In the open-loop 741 case, g_0 is 100,000 and f_c is 10 Hz.

Because you're really only interested in the magnitude of the voltage gain, there's no reason to keep the complex form of **Equation 1**. The magnitude of the voltage gain is given by:

$$|A_v(f)| = \frac{g_0}{[1 + (f/f_c)^2]^{1/2}} \quad (2)$$

Now, if some number of these amplifiers were connected in a multistage amplifier circuit, the midband gain would be increased and the bandwidth reduced as shown in the previous 741 example. Let the number of amplifier stages be n. The magnitude of the voltage gain is then given by:

$$|A_v(f, n)| = \frac{g_0^{\,n}}{[1 + (f/f_c)^2]^{n/2}} \quad (3)$$

Generally, the overall voltage gain is the gain term in which you are interested. Let the midband gain of the *total* amplifier be G_0. Then $G_0 = g_0^n$.

$$|A_v(f, n)| = \frac{G_0}{[1 + (f/f_c)^2]^{n/2}} \quad (4)$$

Upper cutoff frequency

The upper cutoff frequency of the multistage amplifier is that frequency where the total amplifier gain is 3 dB below the midband value. Let the upper cutoff frequency of the total amplifier system be F_c. At that upper cutoff frequency, the voltage gain will be reduced by a factor of $\sqrt{2}$ below the midband value. Thus:

$$|A_v(F_c, n)| = \frac{G_0}{[1 + (F_c/f_c)^2]^{n/2}} = G_0/\sqrt{2}$$

and

$$G_0/\sqrt{2} = \frac{G_0}{[1 + (F_c/f_c)^2]^{n/2}} \quad (5)$$

You can easily solve **Equation 5** for F_c.

$$F_c = f_c \times \sqrt{2^{1/n} - 1} \quad (6)$$

Equation 6 now provides the overall bandwidth (actually the upper cutoff frequency) of a total multistage amplifier of n similar stages. For instance, in the previous example using the 741, if a 741 amplifier were designed for a voltage gain of 17.78, the bandwidth would be 56.2 kHz. If four such amplifiers were used in a multistage amplifier, the overall gain would be $(17.78)^4$, or about 100,000. The bandwidth of the total amplifier is given by **Equation 6** as 24.4 kHz.

A multistage amplifier design example

Suppose you wish to build an amplifier system using one or more amplifier stages with a gain and frequency performance described by **Equation 2**. You want the overall amplifier to have a gain G_0 and the maximum possible bandwidth. The gain-bandwidth product of a single stage is $g_0 \times f_c$. The single-stage gain-bandwidth product is k and constant for any gain.

The 741 example showed that a much wider bandwidth may be obtained for any given gain by using several stages. You may use n stages to provide the total desired gain G_0. The gain of each stage, g_0, must then be $G_0^{\frac{1}{n}}$. So,

$$k = g_0 \times f_c \quad (7)$$

$$G_0 = g_0^n \quad (8)$$

For your overall amplifier, you've specified a specific gain and the maximum possible bandwidth. In other words, you want the maximum possible gain-bandwidth product. The total amplifier gain-bandwidth product is simply $G_0 \times F_c$. You must find this product in terms of the individual stage parameters. You've found G_0 in terms of g_0 in **Equation 8**. But G_0 is a constant—the overall gain that you want. You also know F_c in terms of f_c by **Equation 6**. Then, the gain-bandwidth product of your overall amplifier in terms of the overall gain and the bandwidth of the individual stage is given by:

$$G_0 \times F_c = G_0 \times f_c \times \sqrt{2^{1/n} - 1} \quad (9)$$

It's convenient to put **Equation 9** into a somewhat different form. The radical term is a bit troublesome, but you can make an approximation to improve it. The natural logarithm of a number x, Ln x, may be expressed in an infinite series by **Equation 10**:[2]

$$Ln\, x = (x - 1) - \frac{1}{2}(x - 1)^2 + \frac{1}{3}(x - 1)^3 - \cdots \quad (10)$$

$$Ln\, 2^{1/n} = (2^{1/n} - 1) - \frac{1}{2}(2^{1/n} - 1)^2 + \frac{1}{3}(2^{1/n} - 1)^3 - \cdots \quad (11)$$

If you choose a number of stages for n greater than about 4, the first term of **Equation 11** dominates and the approximation of **Equation 12** is accurate to better than 10 percent. The accuracy improves with more stages.

$$(2^{1/n} - 1) \approx Ln\, 2^{1/n} = \frac{1}{n} Ln\, 2 \quad (12)$$

Equation 9 may then be written as:

$$G_0 \times F_c = G_0 \times f_c \times \sqrt{\frac{1}{n} Ln\, 2} \quad (13)$$

Because you're trying to find the best gain for each stage, you must express f_c and n in terms of g_0. Find this using **Equations 7** and **8**, respectively.

$$f_c = \frac{k}{g_0} \quad (14)$$

$$n = \frac{Ln\, G_0}{Ln\, g_0} \quad (15)$$

By substituting all that information into **Equation 13**, you'll obtain the results of **Equation 17**.

$$G_0 \times F_c = G_0 \left(\frac{k}{g_0}\right) \left[\left(\frac{Ln\, g_0}{Ln\, G_0}\right) (Ln\, 2) \right]^{1/2} \quad (16)$$

$$= G_0 \times k \left(\frac{Ln\, 2}{Ln\, G_0}\right)^{1/2} \left(\frac{Ln\, g_0}{g_0^2}\right)^{1/2} \quad (17)$$

Gain-bandwidth product

Equation 17 now gives the overall amplifier gain-bandwidth product in terms of the various constants and g_0. The term g_0 is the only variable in **Equation 17**. It might appear that G_0 is a variable because it may be expressed in terms of g_0. However, G_0 is the overall total gain desired in the multistage amplifier and is therefore a defined constant. Actually, g_0 is a function of G_0. If you examine **Equation 17**, you'll see that if g_0 is chosen as unity, the gain-bandwidth product will be zero (a unity-stage gain implies an infinite number of stages needed). Similarly, if a very large g_0 is chosen, the gain-bandwidth product also approaches zero (try a gain of 1,000,000). However, if you choose a value of n such as 10, the gain-bandwidth product will be greater than with either unity gain or very high gain. This implies that there is a specific gain which will provide the highest possible gain-bandwidth product. Let that optimum stage gain be g_{opt}. This optimum gain is relatively easy to compute using calculus.[3] However, because not everyone is interested in all the mathematical details, that work is relegated to a sidebar.

After going through the math, the optimum gain g_{opt} and the optimum number of stages n are given in **Equations 18** and **19**.

$$g_{opt} = e^{1/2} = 1.649 = 4.34dB \quad (18)$$

$$n = 2\, Ln\, G_0 \quad (19)$$

$$F_c = f_c \sqrt{\frac{1}{n} Ln\, 2}$$

[from Equations (6) and (12)] **(20)**

Equations 18 and **19** give the optimum stage gain for maximizing gain-bandwidth product and the number of stages needed for any desired gain of a multistage amplifier.

There's a very important but subtle point you should observe in **Equation 18**. Note that the optimum gain of the individual stage is totally independent of everything! It's not a function of the overall gain desired, or the stage bandwidth, or the stage gain-bandwidth product, or anything else except the constant e—the base of the natural logarithm = 2.718. Therefore, if you wish to build an amplifier with the best possible gain-bandwidth product, you'll need a multistage amplifier with individual stage gains of about 1.65—no matter what overall gain you may desire.

Consider the 741 again

If you want to use 741s to build an amplifier with a gain of 100,000 and maximum bandwidth, **Equation 18** shows that the individual stage gain should be 1.649. With that gain, a 741 would provide a bandwidth of about 607 kHz. **Equation 19** indicates that a total of 23 stages would be required. Finally, **Equation 20** shows that the overall bandwidth would be 105 kHz. The overall gain-bandwidth product would be 10.5 GHz. This is considerably better than the 1 MHz gain-bandwidth product of a single 741, although the same voltage gain of 100,000 is provided. I know this may seem a bit peculiar, but just try to obtain a 100-kHz bandwidth with a gain of 100,000 using 741s in any other configuration.

RF amplifier

Even though the 741 example is accurate, it's not a particularly practical or a commonly needed configuration. Consider an RF amplifier. Suppose you have a basic 20-dB gain (voltage gain of 10) amplifier with a 1-GHz bandwidth, 10 Ghz gain-bandwidth product, with the gain adjustable down from 20 dB such that the gain-bandwidth product remains constant. Say you wish to build a multistage amplifier with a gain of 40 dB (voltage gain of 100), but with the best possible bandwidth. If you simply use two of the amplifiers in a two-stage amplifier configuration, you'll have a 40-dB gain and a bandwidth of 644 MHz. However, if you were to use nine stages (nearest integer number to the results of **Equation 19**), each designed for a voltage gain of 1.668, or about 4.44 dB, the overall voltage gain would also be 100. But the bandwidth would be 1.7 GHz, almost a factor of three greater than a simple two-stage unit.

Even a simple 20-dB amplifier can be improved. A single stage of the amplifier above provides a bandwidth of 1 GHz. If five stages, each with a gain of about 1.585 (4.0 dB) were used in a multistage amplifier, the gain would still be 20 dB, but the bandwidth would be slightly greater than 2.3 GHz. That's more than an octave increase in bandwidth.

Conclusions

It's often desirable to optimize the gain and bandwidth of a multistage amplifier to achieve the maximum possible gain-bandwidth product. The optimum voltage gain of each stage of such an amplifier, with at least four stages or more, is $\sqrt{e}$, or approximately 1.65. That stage gain is totally independent of any of the parameters of the individual amplifier stages, or those desired of the multistage unit. Therefore, when designing a multistage amplifier, you should design each stage for a voltage gain of about 1.6 to 1.7 with sufficient stages to provide the desired gain.

You may wish to use prefabricated amplifier building blocks for the individual stages. Typical examples of such building blocks are the MAR devices from MiniCircuits, the GPD and MSA devices from Avantek, and there are numerous others. Although you typically can't modify the gain of these building blocks effectively, there's generally a selection of gain and bandwidths

CALCULATING THE OPTIMUM GAIN

$$G_0 \times F_c = G_0 \times k \left(\frac{Ln\,2}{Ln\,G_0}\right)^{1/2} \left(\frac{Ln\,g_0}{g_0^{\,2}}\right)^{1/2} \quad \mathbf{(17)}$$

G_0 = Desired overall gain = constant

$n > \approx 4$

Limit $G_0 \times F_c = 0$ and Limit $G_0 \times F_c = 0$

$g_0 \to 0 \qquad\qquad g_0 \to \infty$

$$For\ 0 \le g_0 \le \infty,\ G_0 \times F_c > 0$$

Therefore, a maximum must exist. To find the maximum, take the first derivative of $G_0 \times F_c$ with respect to g_0, equate to zero, and solve for $g_0 \equiv g_{opt}$.

$$\frac{d}{dg_0}(G_0 \times F_c) =$$

$$G_0 \times k \left(\frac{Ln\,2}{Ln\,G_0}\right)^{1/2} \frac{d}{dg_0}\left(\frac{Ln\,g_0}{g_0}\right)^{1/2} \equiv 0$$

$$(Ln\,g_0)^{-1/2}(g_0 - 2g_0\,Ln\,g_0) = 0$$

$$(Ln\,g_0)^{-1/2}(1 - 2Ln\,g_0) = 0 \quad [for\ g_0 \neq 0]$$

$$1 - 2Ln\,g_0 = 0 \quad [for\ Ln\,g_0 \neq 0]$$

$$Ln\,g_0 = \frac{1}{2}$$

$$g_0 \equiv g_{opt} = e^{1/2}$$

$$G_0 = g_{opt}^{\,n} \quad \mathbf{(18)}$$

$$n = \frac{Ln\,G_0}{Ln\,g_{opt}} = 2\,Ln\,G_0 \quad \mathbf{(19)}$$

offered in each series of devices. The lower-gain devices usually provide the wider bandwidth. If you wish to maximize the bandwidth for any desired gain in a multistage amplifier, it's important to select the individual amplifier device of the group with the *lowest* gain, and presumably, widest bandwidth.

Although the optimum gain-bandwidth product of a multistage amplifier is obtained with a stage gain of $\sqrt{e}$, there may be other circumstances where you may prefer a higher gain in a specific stage. One situation where this is true is in low-noise designs. In these cases, it's very important that the first stage have a relatively high gain to prevent compromise of the amplifier noise figure by the noise of succeeding stages. You may also prefer higher gain in a specific stage in high-output power amplifiers. A final-stage gain of only 4 dB may be insufficient to allow the maximum output of the preceding stage to drive the final stage to its maximum output capability. In this case, a higher-gain final would be desirable. Therefore, in specific cases, a gain higher than the optimum for best gain-bandwidth product may be required. However, when such a higher gain is used, it should be understood that it's a tradeoff of gain-bandwidth product for improvement in some other parameters. ■

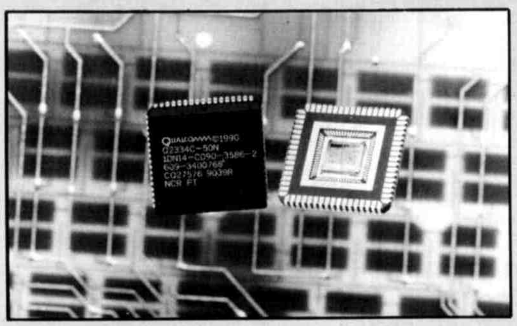
REFERENCES

1. P.M. Chirlian, *Analysis of Design of Electronic Circuits*, McGraw Hill, New York, 1965.
2. M. Abramowitz and I.R. Stegun, *Handbook of Mathematical Functions*, Ninth Printing, U.S. Government Printing Office, Washington, 1970.
3. E.J. Johnson and F.L. Kiokemeister, *Calculus*, Allyn and Bacon, Inc., Boston, 1959.

PRODUCT INFORMATION

New 20-MHz Direct Digital Synthesizer

QUALCOMM, Inc. announces a 20-MHz dual Direct Digital Synthesizer (DDS), the Q2334M-20L, which provides two independent synthesizers on one integrated circuit for military applications. The Q2334 provides output over a wide bandwidth and generates two independent signals for completely separate circuit functions (i.e., complex signal generation, I/Q channels). The device includes two patented features: a noise reduction circuit, which lets the user specify less expensive DACs without the expected increase in spur levels; and an algorithmic sine lookup.



Packaged in a 68-pin hermetically sealed ceramic leaded chip carrier (CLDCC), the Q2334M-20L is screened to the requirements of MIL-STD-883, Level B techniques of Methods 5004. QUALCOMM has also released a commercial ceramic version of the 20-MHz DDS (part no. Q2334I-20L) and the 30-MHz DDS (part no. Q2334I-30L).

For further information contact QUALCOMM, Inc., VSLI Products Division, 10555 Sorrento Valley Road, San Diego, California 92121-1617.

New High Voltage Relay Available

Kilovac Corporation introduces a new high voltage relay. The HC-6 is a new pressurized gas-filled relay with tungsten and molybdenum contacts, and a rated operating voltage of 8 kV. The relay's continuous current carry is 15 amps and, under certain circumstances, it can make up to 150 amps.

For more information on the HC-6, contact Kilovac Corporation, P.O. Box 4422, Santa Barbara, California 93140.