# OPTIMUM NOISE-MATCH SOURCE RESISTANCE

M. E. Gruchalla, June 29, 2006

**Abstract**

**The concept of optimum noise matching a resistive source to an amplifier exhibiting equivalent input noise voltage and current sources is reviewed. Noise models of the source resistance and the amplifier are presented. The optimum source resistance for lowest-possible noise factor and highest possible signal-to-noise ratio is derived.**

## Introduction

All amplifiers exhibit noise, and all resistive sources exhibit thermal noise by virtue of temperature. When a source having a real resistive component is connected to an amplifier input as shown in Figure 1, the output noise includes contributions from both the amplifier and the source resistance. For any given amplifier, there exists an optimum source resistance that will provide the lowest noise factor and in turn the highest signal-to-noise ratio for any specific signal. This paper derives the optimum source resistance required to optimize the noise performance of an amplifier.

For this work, only first-principle noise sources are considered. All noise is defined as Gaussian, white, stationary and ergodic, and all independent noise sources are defined as uncorrelated. For convenience and generality, the spectral density of each noise source is used in all computations. Total noise contribution of each source is computed by integrating the noise-spectral-density contribution of each source over the response characteristic of the system. If the system is a well-behaved first or second-order system the noise bandwidth may be used as a convenient parameter to eliminate the need for this integration. The noise bandwidth for first-order systems is a factor of $\pi/2$ greater than the signal bandwidth, and a factor of 1.22 for second-order systems.

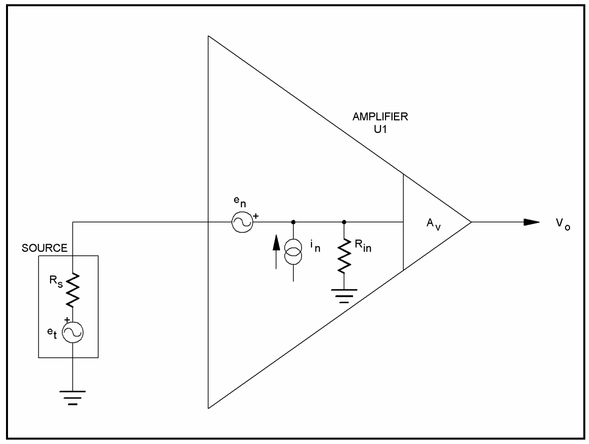


**Figure 1. Noise-Analysis Model**

Specifically excluded from this review are all forms of excess noise and cultural noise. Such noise sources are not first-principle processes and typically cannot be described statistically. Accordingly, excess noise cannot be accurately predicted or computed. In any low-noise design, all excess and cultural noise must be eliminated if performance limited by first-principle processes is to be achieved.

Also, $1/f$ noise is problematic in applications where the low-frequency response of the system is below the $1/f$ noise corner of the noise sources. $1/f$ noise is poorly understood and is generally a type of excess noise. If such low-frequency operation is required, some form of parametric amplifier topology will likely provide improved noise performance.

## General Noise Model

The circuit model to be analyzed is shown in Figure 1. This model comprises a source of resistance $R_s$ applied to the input of amplifier $U1$. Both the source resistance and the amplifier are modeled as real devices compromising noise where the noise is modeled as described below. This model is defined as linear and therefore superposition may be applied.

## Resistor Noise Model

The noise model for the source resistance used for this analysis is the canonical resistor noise model shown in Figure 1. This model for a real resistive source compromises a noiseless resistor of value $R_s$ in series with the thermal noise voltage of that resistance, $e_t = \sqrt{4kTR_s}$ . This is the available thermal noise voltage per unit bandwidth available from the source resistance $R_s$.

## Amplifier Noise Model

The amplifier noise model used for this analysis is also the canonical amplifier noise model shown in Figure 1. The amplifier noise model comprises an amplifier of ideal noiseless voltage gain $A_v$, ideal noiseless input resistance $R_{in}$, equivalent input noise current source $i_n$ and equivalent input voltage source $e_n$. It is important to understand that all noise is modeled into the two equivalent input noise sources, and both the input resistance and the amplifier gain are defined as noiseless.

Examining Figure 1, it is easily seen that for an input open-circuit condition, all current $i_n$ flows in input resistance $R_{in}$, is amplified by amplifier gain $A_v$ and appears at the amplifier output as a noise voltage, but there is no output noise contribution from $e_n$ since one terminal of $e_n$ is floating in the open-circuit configuration. Similarly, for an input short-circuit condition, the source $e_n$ is amplified by $A_v$ and appears at the amplifier output as a noise voltage, but there is no contribution from $i_n$ since all $i_n$ is shunted through the input short circuit. However, if a resistive source is connected at the amplifier input as in Figure 1, the total amplifier output noise is not as obvious. In such a configuration, there are three contributions to the total amplifier output noise: a contribution from the source resistance, a contribution from the equivalent input noise voltage source and a contribution from the equivalent input current source.

## Noise Figure-of-Merit

A specific figure-of-merit must be defined in order to measure the relative noise performance of any system. For this analysis, the Noise Factor, $F$ , is used as the figure of merit to be optimized. Optimizing the noise factor also maximizes the signal-to-noise ratio for any specific signal.

The noise factor is defined as the total output noise power, including both the contributions from the amplifier and the source itself, divided by the output noise power due to the source alone. It is important to note that the noise factor is defined in terms of power and is specified at the output of the system.

## Amplifier Noise Analysis

The simplicity of the model of Figure 1 permits it to be analyzed primarily by inspection. As noted, there are three output noise components to be computed. Since the model is defined as linear, superposition may be applied. However, superposition must be applied to the mean-square values of these noise components since these are statistical quantities. Also, since all the noise sources are defined to be uncorrelated, the mean-square components may be summed without consideration of a correlation coefficient. Therefore, the mean square value of each of the components may be computed separately and simply combined by superposition at the output to provide the total output noise.

The mean-square output noise voltage due to the source resistance is easily written by inspection.

$$V_o(R_s)^2 = (4kTR_s)\cdot\left(\frac{R_{in}}{R_{in}+R_s}\right)^2 \cdot A_v^{\,2} \tag{1}$$

where $V_o(R_s)$ denotes that $V_o$ is a function of $R_s$.

Similarly, the mean-square output noise contributions due to the equivalent input voltage and current noise sources are also easily written by inspection.

$$V_o(e_n)^2 = (e_n)^2\cdot\left(\frac{R_{in}}{R_{in}+R_s}\right)^2 \cdot A_v^{\,2} \tag{2}$$

$$V_o(i_n)^2 = (i_n)^2\cdot\left(\frac{R_s}{R_{in}+R_s}\right)^2 \cdot R_{in}^{\,2}\cdot A_v^{\,2} \tag{3}$$

The noise factor may be written directly from Equations 1, 2, and 3.

$$F \equiv \frac{P_{no}[Total]}{P_{no}[Source]} \tag{4}$$

Since the output power delivered is used in the noise factor expression, the output load resistance $R_L$ must be included.

$$F = \frac{(4kTR_s)\cdot\left(\frac{R_{in}}{R_{in}+R_s}\right)^2\cdot A_v^{\,2}\cdot R_L + e_n^{\,2}\cdot\left(\frac{R_{in}}{R_{in}+R_s}\right)^2\cdot A_v^{\,2}\cdot R_L + i_n^{\,2}\cdot\left(\frac{R_s}{R_{in}+R_s}\right)^2\cdot R_{in}^{\,2}\cdot A_v^{\,2}\cdot R_L}{(4kTR_s)\cdot\left(\frac{R_{in}}{R_{in}+R_s}\right)^2\cdot A_v^{\,2}\cdot R_L} \tag{5}$$

As a sanity check, checking the units of all the terms of Equation 5 confirms that the ratio is dimensionless. The expression of Equation 5 may be simplified substantially.

$$F = \frac{(4kTR_s) + e_n^{\,2} + i_n^{\,2}\cdot R_s^{\,2}}{(4kTR_s)} \tag{6}$$

It is instructive to examine Equation 6. The noise factor in the limit as the source resistance is allowed to approach zero is infinite. And the limit is also infinite as the source resistance is allowed to approach infinity. But for a finite, non-zero value of source resistance, the value of the noise factor is finite. Therefore, there is at least one value of source resistance that will yield a minima in the noise factor. Since the system is effectively first order, there is only a single value of source resistance that will yield such a minimum value. This is the lowest possible noise factor. That value of optimum source resistance, $R_{sopt}$, may be easily found by taking the partial derivative of the noise factor with respect to $R_s$, equating it to zero to find the inflection point, and solving for the optimum source resistance.

$$\frac{\partial F}{\partial R_s} = \frac{i_n^{\,2}}{4kT} - \frac{e_n^{\,2}}{4kTR_s^{\,2}} \tag{7}$$

$$\frac{i_n^{\,2}}{4kT} - \frac{e_n^{\,2}}{4kTR_s^{\,2}} \equiv 0 \tag{8}$$

$$R_s \equiv R_{sopt} = \frac{e_n}{i_n} \tag{9}$$

Equation 9 provides the value of source resistance that will yield the best-possible noise factor and the highest-possible signal-to-noise ratio. It should be noted that Equation 9 may be applied either with the noise spectral density to provide an optimum source resistance at a specific frequency, or it may be applied over a bandwidth were the noise sources are the total integrated noise over the system response bandwidth.

The optimum noise factor may be found by simply substituting the value for $R_s$ given in Equation 9 into Equation 6.

$$F_{opt} = \frac{\left(4kT \cdot \frac{e_n}{i_n}\right) + e_n{}^2 + i_n{}^2 \cdot \left(\frac{e_n}{i_n}\right)^2}{\left(4kT \cdot \frac{e_n}{i_n}\right)} \qquad (10)$$

$$F_{opt} = \frac{4kT + 2 \cdot e_n \cdot i_n}{4kT} \qquad (11)$$

Equation 11 provides the value of the best-possible noise factor that may be achieved with the specific amplifier and its associated equivalent input noise sources. And this optimum noise factor occurs only for the optimum source resistance given by Equation 9.

An important fact to consider is that the noise factor is a power ratio. The equivalent voltage or current ratio is given by the square root of the noise factor. Accordingly, when applying the noise factor as given by Equation 11 to compute the total output noise voltage or current, the output noise voltage due to the source alone must be multiplied by $\sqrt{F_{opt}}$.

### Interpretation

The result of Equation 9 is rather provocative. This shows that the optimum source resistance to provide the best possible noise factor, and in turn the highest-possible signal-to-noise ratio, is a function of only the equivalent input noise sources of the amplifier. Specifically, it is independent of the amplifier input resistance and the amplifier gain. This is not immediately obvious from examination of Figure 1.

Also as provocative is Equation 11. This shows that the best possible noise factor too is a function only of the amplifier equivalent input noise sources. It is instructive to note that the noise factor is a function of the equivalent input noise power of the amplifier defined as the product of the equivalent input noise voltage and current sources. In Equation 11, the first term is recognized as the background thermal noise power. In Equation 11, it is seen that the effective noise power added to the thermal background noise power is twice the amplifier equivalent input noise power. For any given amplifier, this is the best possible noise performance that may be achieved, and this optimum occurs only for the optimum source resistance given by Equation 9.

### Importance of the Optimum Power-Match Resistance

Often it is assumed that the best noise performance will be provided when the source is optimally power matched to the input resistance of the amplifier. The argument is that at that point the maximum power is transferred to the amplifier input, and it is argued that with maximum power transfer to the amplifier, the best noise performance will be achieved. This is a serious error, but one that is very common. Equation 9 clearly shows that the optimum source resistance is totally independent of the amplifier input resistance. Therefore, the optimum power match resistance has no significance in optimizing overall noise performance.