Q

M. E. Gruchalla, November 7, 1997

One of the very common parameters that we use in rf applications is Q, or “Quality Factor.” But, just what is this parameter Q? This brief paper derives Q in terms of typical circuit parameters based on the fundamental definition of the Quality Factor. A few examples of the proper and improper common uses of Q are then reviewed. Don’t let the math bother you too much; it is just included to show you were the formulae are derived.

Q is a parameter used to measure “how good” a resonant circuit is — in other words, how well it oscillates, or resonates. So our first useful piece of information is that Q applies only to a resonant network. For example, a clock pendulum is an example of a simple resonator. If the pendulum is pulled back to some point, potential energy is “stored” in the pendulum since it is raised slightly above its rest position. When the pendulum is released, it “oscillates” back and forth. At the exact bottom of its swing, the pendulum is at its lowest point, point of zero potential energy relative to its path of motion, and it has maximum velocity at this point. All the potential energy stored at the top of the swing where it was released is exchanged for “kinetic” energy at the bottom of the swing. At the bottom of the swing, the energy is “stored” in the pendulum as “kinetic” energy. The pendulum of course will not swing forever if not driven in some manner. This is due to losses, or resistance, in the mechanical system, such as air drag and friction loss in the suspension mechanism of the pendulum. These losses cause the amplitude of the pendulum to gradually diminish, or “ring down” as it is commonly referred to for a resonant network. The higher the losses, the quicker the pendulum amplitude diminishes. Q is a measure of how low the losses are and how long a resonator such as a pendulum will swing — the lower the losses the higher the Q and the slower the pendulum amplitude will diminish.

Now the very subtle but very important point to get from this example is that the parameter Q applies only to a very special type of network: a resonant network that contains two different types of energy storage and some type of loss. In the mechanical pendulum system, the two energy storage elements are the potential energy of the height of the pendulum and the kinetic energy of the velocity of the pendulum, and the loss element is all of the friction losses of the system. In an electrical network, one can make the analogy (as well as others) of potential energy being the charge in a capacitor. This is a somewhat logical analogy since when a capacitor is charged, there is nothing moving, so it corresponds logically to pulling the pendulum up to some position and holding it there. Similarly, energy is stored in an inductor by a flow of current in the inductor, so a flowing inductor current can be thought of as being equivalent to the kinetic energy in the moving pendulum. The system losses are modeled as a resistance in the electrical circuit. So, in an electrical circuit, the system energy actually moves from one circuit element to another, back an forth between the inductor and capacitor, and the energy stored in the system is slowly dissipated be the resistance. This is a little easier to see than the pendulum example. But again the important thing to understand is that Q only applies to a circuit with one inductor, one capacitor and one resistor. This network is a second-order system often termed a "simple harmonic oscillator." And, specifically, Q applies only to a second-order network.

The fundamental definition of Q is based on a simple second-order network in terms of its ***undamped natural frequency***. Q is defined as the ratio of the ***total energy stored*** in the resonant network to the ***energy dissipated per radian of the drive signal*** when the network is driven at its undamped natural frequency. In terms of a cycle of the drive signal, Q is defined as 2π times the energy stored divided by the energy dissipated per cycle of the driving source. This is not as complicated as it sounds, however there are some subtle but very important features of this definition.

First a "simple second-order network" as noted above is simply a resonant circuit comprising some inductance, some capacitance, and some resistance. As we will see below, if the losses are zero, the Q will be infinite. Also as in the example above, Q is not limited to electrical circuits.

One may properly compute the mechanical Q of a clock pendulum, or the acoustic Q of an auditorium.

We may configure our harmonic oscillator as a parallel-resonant network or as a series-resonant network. Although both of these configurations have a specific value of Q, the computation of Q is different for the two. For example, in a parallel-resonant circuit, it will be shown below that as the resistance becomes very large the Q becomes very large, but in the series-resonant network, Q becomes very large as the resistance becomes very small. So in the parallel-resonant case, the Q is directly related to the network resistance, but in the series-resonant case it is related to the reciprocal of the resistance. So, when we speak of Q or compute Q for some circuit, we must know what type of circuit we are talking about. The harmonic oscillator part of the definition should be pretty clear, but if is not, just continue on — when we get to the actual calculations, it should clear up a bit more.

Now for the second part of the definition: total energy stored. In a simple harmonic oscillator, such as an R-L- C network, there are two elements that store energy: the inductance and the capacitance (or their equivalent in a mechanical system). The total energy stored in the network at any instant is simply the sum of the energy stored in the inductance and the energy stored in the capacitance. Think about the clock pendulum again. Suppose that it is an “ideal” pendulum without any loss. Then it would swing forever. Now, if it swings forever, it loses no energy. Also, since it always swings the same, in other words the magnitude of its swing does not change from swing to swing, it gains no energy either. So, if no energy leaves the pendulum system and no energy enters it, the energy in the ideal pendulum system must be constant. The energy in the system simply moves between potential energy and kinetic energy, and at any instant in the swing, the total energy in the system is simply the sum of the potential energy of the pendulum by virtue of its height above its minimum position at that instant and the kinetic energy of its velocity at that instant. Since the ideal pendulum neither loses nor gains energy, this sum must be always constant no matter where in the swing the pendulum is.

Suppose that we now take an electrical parallel R-L- C network and drive it with a sinusoidal voltage source. Since we are driving this circuit with a sinusoidal source, the voltage across the capacitor will be sinusoidal, and since the energy stored in the capacitor is proportional to the capacitor voltage squared, the energy stored in the capacitor will go from zero at the zero crossing of the voltage across the capacitor to some value proportional to the peak capacitor voltage squared. Similarly, the inductor current will be sinusoidal as well, and since the energy stored in the inductor is proportional to the inductor current squared, the energy stored in the inductor will vary from zero to some value related to the square of the peak-inductor current. So, it seems that the value of Q that we will get depends on where in the cycle of the drive signal we compute the Q since the energies stored in the capacitor and the inductor seem to depend where we are in the cycle. But, we will show that if we drive the R-L- C network at precisely its undamped natural frequency, which will be defined shortly, the sum of the energy in the inductor and capacitor will always be constant. The total stored energy simply moves back and forth between the inductance and the capacitance. For example, when the voltage across the capacitor is zero, all the energy is in the inductor, and when the inductor current is zero, all the energy is in the capacitor. At any point in between, some of the energy is in the inductor and some in the capacitor, but the total energy is always constant. If we know the amplitude of the driving signal, we can compute the voltage signal across the capacitor and the current through the inductor. We can then state that if we know the drive signal magnitude and frequency, we can compute the inductor and capacitor signals.

Now for the final part of the definition: the energy dissipated per radian, or per cycle, of the driving source. This may bit a little confusing, so huddle around and listen closely. Power is simply the flow of energy. In other words, a specific delivered power means that a given amount of energy passes into a load in some specific time. Energy has the units of watt-seconds (joules), and of course power the units of watts. So if there is some amount of energy in watt-seconds flowing into a load in some specific time, the power is simply the energy that flows into the load in that time divided by the time. For example, if an energy of 100W-s flows into a load in 10s, the power is 100W-s divided by 10s, or 10W. Simple enough. So, a continuous flow of energy per time is a constant power. This may be confusing since we tend to think of power flowing in a circuit, but it is the energy that flows through the circuit resulting in power being delivered at some load. So power P is equal to the energy E delivered over some time T divided by that time T,

$$Power = \frac{Energy}{Time} \Rightarrow P = \frac{E}{T} \tag{1}$$

Remember that the *T* term in Equation (1) is the time over which the energy *E* is delivered.

In order to compute Q, we need to know the energy dissipated per radian, or per cycle, of the driving source. Well, a cycle of the driving source occurs in a specific time, specifically the period of the signal, or one over the frequency of the source signal. Also, in our simple driven harmonic oscillator, if we choose the source right, we will be able to very easily compute the power dissipated in the oscillator. The only element that can dissipate power is the resistive element of the R-L-C network. So, the power dissipated in the simple harmonic oscillator is simply the power dissipated by the resistive element.

Now, Equation (1) gives us the power that is continuously dissipated when a specific flow of energy is delivered to a load over a specific time. So, if we solve for the energy *E* in terms of the power *P* and the time *T*, what we have is the energy required to be delivered to the load in time *T* in order to result in a dissipated power *P*.

$$E = P \cdot T \tag{2}$$

Now, if we know the power dissipation, for any time interval we may compute the energy that must be delivered over that interval in order to achieve that power. Suppose we chose the time interval as that of one cycle of our driving source. Let that time interval be defined as "t_s."

$$t_s = \frac{1}{f_s} \tag{3}$$

So, in order to deliver a power *P*, the energy E_f that must be delivered over a cycle of the source is given by Equation (4).

$$E_f = P \cdot t = \frac{P}{f_s} \tag{4}$$

The energy E_f is delivered over 2π radians of the source, which is one cycle of the source signal. So, if we divide both sides of Equation (4) by 2π, we will have the energy dissipated per radian of the source. Let that energy be E_a for the energy dissipated in terms of the angular frequency (radians per second) of the source, specifically the energy dissipated per radian.

$$E_a = \frac{E_f}{2\pi} = \frac{P \cdot t}{2\pi} = \frac{P}{2\pi f_s} \tag{5}$$

We should now have everything we need to compute Q, well sort of anyway. We of course know the values of the inductance, capacitance and resistance of the network and whether it is a parallel-resonant or series-resonant circuit since those are given parameters. We have a frequency f_s at which we are driving our simple harmonic oscillator, we know the total energy stored in the network since we know the amplitude of the signals at the inductance and capacitance, and we know the energy dissipated per cycle of the driving signal since we know the power being dissipated by the resistive element of the network. So, given only the values of an R-L-C network and its configuration, parallel or series, we may compute the Q of the network, theoretically.

Now let's look at a real circuit. Figure 1 shows a simple parallel R-L-C circuit driven by a voltage source $V(t)$ where

$$V(t) = V_0 Sin(\omega_s t) \tag{6}$$

The term ω_s is simply the angular frequency of the driving source where,

$$\omega_s = 2\pi f_s \tag{7}$$

We have seen this term before — look back at Equation (5). A voltage source is used to drive this configuration since this will simplify the math a bit, but a current source could have just as well been used. By choosing a voltage source, we define the capacitor and resistor voltage so the only signal that must be computed is the inductor current.

First the easy part — computing the continuous power dissipation. The only element in the circuit that dissipates power is the resistance (this assumes that there is no radiation from the inductance and wiring and that there are no other resistances in the circuit). Notice that since the source was conveniently chosen as a voltage source driving the parallel network, this voltage source is simply connected across the resistor. When we drive a resistance with a sinusoidal voltage source of peak value V_0, the power dissipation is simply one half the square of the peak voltage divided by the resistance. The fact that the inductance and capacitance are also connected across the source is of no consequence since at the undamped natural frequency, the impedance of a parallel-resonant L-C network is infinite. The power dissipated by the resistor is simply related to the voltage across it squared.

$$P = \frac{V_0^{\,2}/2}{R} = \frac{1}{2}\frac{V_0^{\,2}}{R} \tag{8}$$

The next part is a bit more complicated — finding the total stored energy (there is a very easy way to do this, but we will do it the hard way just so everyone will believe the easy way). The capacitor first. The energy in the capacitor at any instant in time is simply one half the capacitor voltage squared multiplied by the capacitance. Suppose that we begin driving our network at the zero crossing of the driving source and we define that time as zero time. Then at a later time t_1 sufficiently long that all transients have died away we look at the voltage across the capacitor.

$$V_c(t_1) = V_0 Sin(\omega_s t_1) \tag{9}$$

At that time t_1 the energy $E_c(t_1)$ stored in the capacitor is simply,

$$E_c(t_1) = \frac{1}{2}C\left[V_c(t_1)\right]^2 = \frac{1}{2}CV_0^{\ 2} Sin^2(\omega_s t_1) \tag{10}$$

Now the inductor. The energy stored in the inductor is one half the inductor current squared multiplied by the inductance.

$$E_L = \frac{1}{2}LI_L^{\ 2} \tag{11}$$

The inductor voltage is equal to the inductance times the time derivative of the inductor current. So, the inductor current is the integral of the inductor voltage divided by the inductance.

$$I_L = \frac{1}{L}\int V_L(t)dt = \frac{1}{L}\int V_0 Sin(\omega_s t)dt \tag{12}$$

$$I_L = -\frac{V_0}{\omega_s L} Cos(\omega_s t) \tag{13}$$

The Cosine term in the inductor current shows that the inductor current is 90°, or $\pi/2$ radians, out of phase with respect to the inductor voltage, and the minus sign shows that it is “lagging” the voltage — in other words, when the voltage reaches a peak across the inductor, the inductor current will reach its peak 90° later.

Therefore, at time t_1 the energy in the inductor is given by Equation (14).

$$E_L(t_1) = \frac{1}{2}L\left[I_L(t_1)\right]^2 = \frac{1}{2}L\left[-\frac{V_0}{\omega L} Cos(\omega_s t_1)\right]^2$$

$$E_L\left(t_1\right)=\frac{V_0{}^2 Cos^2\left(\omega_s t_1\right)}{2\omega_s{}^2 L} \tag{14}$$

The total energy E_T in the network at any time t_1 is simply the sum of Equations (10) and (14).

$$E_T\left(t_1\right)=\frac{CV_0{}^2 Sin^2\left(\omega_s t_1\right)}{2}+\frac{V_0{}^2 Cos^2\left(\omega_s t_1\right)}{2\omega_s{}^2 L}$$

$$=\frac{1}{2}V_0{}^2\left(CSin^2\left(\omega_s t_1\right)+\frac{1}{\omega^2 L}Cos^2\left(\omega_s t_1\right)\right) \tag{15}$$

Equation (15) shows that in general the total energy in the network is a function of time — as t_1 changes E_T changes. This is very important to note.

We have not yet defined ω_s. As noted above, it is simply 2π times the driving frequency, but we have not yet selected a driving frequency. Suppose that we ***define*** a network-related frequency ω_0 by Equation (16).

$$\omega_0{}^2\equiv\frac{1}{LC} \tag{16}$$

In other words, for a given inductance and capacitance of a parallel R-L-C network, the frequency ω_0 is defined by Equation (16). In network theory, this frequency is termed the ***undamped natural frequency*** of the network. This however is not the resonant frequency — the resistance element of the network causes the resonant frequency to be shifted from the undamped natural frequency. The resonant frequency is termed the ***natural frequency*** of the network. We will calculate the natural frequency later in terms of Q to show how the natural frequency is shifted from the undamped natural frequency by the network Q.

Now suppose that we drive the network at precisely the undamped natural frequency, in other words we make $\omega_s = \omega_0$. First, substitute ω_0 for ω_s in Equation (15).

$$E_T\left(t_1\right) = \frac{1}{2}V_0{}^2\left(CSin^2\left(\omega_0 t_1\right) + \frac{1}{\omega_0{}^2 L}Cos^2\left(\omega_0 t_1\right)\right) \tag{17}$$

Then substitute the value for $\omega_0{}^2$ form Equation (16) in the coefficient of the Cosine term in Equation (17).

$$E_T\left(t_1\right) = \frac{1}{2}V_0{}^2\left(CSin^2\left(\omega_0 t_1\right) + \frac{1}{\left(\frac{1}{LC}\right)L}Cos^2\left(\omega_0 t_1\right)\right)$$

$$= \frac{1}{2}CV_0{}^2\left[Sin^2\left(\omega_0 t_1\right) + Cos^2\left(\omega_0 t_1\right)\right]$$

But,

$$\left[Sin^2\left(\omega_0 t_1\right) + Cos^2\left(\omega_0 t_1\right)\right] = 1$$

So,

$$E_T\left(t_1\right) = \frac{1}{2}CV_0{}^2 \tag{18}$$

Now that is an interesting result. The total energy stored in the network is independent of time if the system is driven at its undamped natural frequency ω_0 as defined in Equation (16), and is equal to one half the capacitance times the peak magnitude of the voltage of the driving source squared. This was the easy way of finding the total stored energy mentioned above — simply the “potential energy” stored in the capacitor at the peak of the voltage swing. But here we actually derived it so that we can see where it comes from. When the voltage across the capacitor, and the inductor and resistor as well, is at its peak value, the inductor current is zero so all of the total stored energy is in the capacitor. Similarly, when the voltage is zero, the current in the inductor will be at its peak value. The total energy will then be one half the inductance times the peak inductor current squared. This can be shown by solving Equation (16) for C and substituting that value into Equation (15).

$$C = \frac{1}{\omega_0{}^2 L} \tag{19}$$

$$E_T = \frac{1}{2} C V_0{}^2 = \frac{1}{2}\left(\frac{1}{\omega_0{}^2 L}\right) V_0{}^2 = \frac{1}{2} L \left(\frac{V_0{}^2}{\omega_0{}^2 L^2}\right)$$

$$E_T = \frac{1}{2} L \left(\frac{V_0}{\omega_0 L}\right)^2 \tag{20}$$

From Equation (13) it is seen that the squared term in Equation (20) is simply the peak value of the inductor current. Therefore, the total energy E_T is constant and given by Equation (21)

$$E_T = \frac{1}{2} C V_0{}^2 = \frac{1}{2} L I_0{}^2 \tag{21}$$

The total energy in the simple harmonic oscillator may be computed as either one half the capacitance times the square of the peak capacitor voltage or one half the inductance times square of the peak inductor current. In the case of our parallel network driven by a voltage source, we always know the capacitor voltage since we are driving it directly with the source. So in the case of the parallel network, it is most convenient to compute the total energy of the network using the capacitive form for the total energy given by Equation (21).

Finally before we are actually ready to compute Q we must compute the energy dissipated per cycle of the driving source. We can find this from Equations (5) and (8),

$$E_a = \frac{P}{2\pi f_0} = \frac{1}{2}\frac{V_0^{\ 2}}{2\pi f_0 R}$$

We are now ready to actually compute Q. Let Q_p represent the Q of a parallel-resonant circuit.

$$Q_p = \frac{E_T}{E_a} = \frac{\frac{1}{2}CV_0^{\ 2}}{\frac{1}{2}\frac{V_0^{\ 2}}{2\pi f_0 R}} = 2\pi f_0 RC = \omega_0 RC \tag{22}$$

We can also find Q in terms of the inductance. Substituting Equation (19) into Equation (22).

$$Q_p = \omega_0 R\left(\frac{1}{\omega_0^{\ 2} L}\right) = \frac{R}{\omega_0 L} \tag{23}$$

Now we notice in Equation (22) that the value of Q is given in terms of the capacitance and ω_0, and in (23) in terms of the inductance and ω_0. Suppose that we solve Equation (22) for ω_0 and substitute it into Equation (23).

$$Q_p = R\sqrt{C/L} \tag{24}$$

This result is a bit more informative. Notice that in this form, Q is a function of R, L and C, but is not at all a function of frequency. This demonstrates that Q is truly a network parameter and function only of the circuit parameters, and specifically Q is not a function of the frequency at which the network is operating. Q is just as much a network parameter as the values of the network elements, the $R \cdot C$ time constant or the L/R time constant. This is an important observation. Also, since the term $\sqrt{C/L}$ has the units of one over resistance (or $\sqrt{L/C}$ has the units of resistance), Q is dimensionless.

So, summarizing the parallel R-L-C network case,

$$Q_p = \omega_0 RC = \frac{R}{\omega_0 L} = R\sqrt{C/L} \tag{25}$$

Now, consider a series-resonant circuit such as shown in Figure 2. Here the series-resonant configuration is driven by a current source for convenience just as the parallel circuit was driven with a voltage source. By driving with a current source, we precisely know the current in each element. This makes it very convenient to compute the power dissipated in the resistor and the total energy stored from the inductor peak current.

The power dissipation is simply one half the square of the peak current times the resistance.

$$P = \frac{1}{2} I_0{}^2 R \tag{26}$$

And the energy dissipated per cycle,

$$E_f = \frac{P}{f_0} = \frac{1}{2}\frac{I_0{}^2 R}{f_0} \tag{27}$$

And the energy dissipated per radian,

$$E_a = \frac{P}{2\pi f_0} = \frac{1}{2}\frac{I_0{}^2 R}{2\pi f_0} \tag{28}$$

Using the same approach as above we can show that the total energy stored in the system is again equal to one half the inductance times the square of the peak inductor current, and of course one half the capacitance times the square of the peak capacitor voltage. So, we will not derive this for the for the series network — for those of you that are interested (or don't believe it), this derivation is "left as an exercise to the reader," to put it in academic terms.

In the case of the series circuit driven by a current source, we know the inductor current so the energy expressed in terms of the inductance is the most useful here.

$$E_T = \frac{1}{2} L I_0^{\,2} \tag{29}$$

Let Q_s be the Q of the series circuit.

$$Q_s = \frac{E_T}{E_a} = \frac{\frac{1}{2} L I_0^{\,2}}{\frac{1}{2}\frac{I_0^{\,2} R}{2\pi f_0}} = \frac{2\pi f_0 L}{R} = \frac{\omega_0 L}{R} \tag{30}$$

Solving Equation (16) for L and substitution into Equation (30),

$$Q_s = \frac{\omega_0}{R}\left(\frac{1}{\omega_0^{\,2} C}\right) = \frac{1}{\omega_0 R C} \tag{31}$$

Then solving for ω_0 in Equation (30) and substituting into Equation (31),

$$Q_s = \frac{1}{R}\sqrt{L/C} \tag{32}$$

Then, summarizing for the series case,

$$Q_s = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC} = \frac{1}{R}\sqrt{L/C} \tag{33}$$

So there we have it, the Q for both a parallel resonant network and a series resonant network, and they are not the same. Interesting. Equations (24) and (32) are perhaps the most important of all those presented in this paper. These show that the Quality Factor Q of a network it a true network parameter, and that Q is totally independent of the actual operating frequency of the network. Q is a ***defined*** network parameter defined totally in terms of the network element values, that is the values of the network R, L, and C.

Let's think about what we have discovered here. First, and very very important, we have defined Q in terms of a very specific frequency — the undamped natural frequency of the network. If we use any other frequency, Equation (15) shows that the Q value is then a function of time and is not a constant value based only on the network component values. But if we define the Q in terms of the undamped natural frequency, given by Equation (16), the value of Q becomes independent of time and dependent only on the circuit values. This is shown explicitly in Equations (24) and (32). The Q, or Quality Factor, is a general measure of the *quality* of a resonant network, or just how well it resonates and how slow the network *rings down* when excited. It is simply a measure of the loss in the network — the higher the losses the faster the network will ring down and the lower the Q. To be a universal general network parameter, it is most convenient to define Q totally in terms of the network parameters alone. Therefore, the specific frequency for computing Q from the fundamental definition of Q is ***defined*** as the undamped natural frequency of the network. So, Q is simply another network parameter such as the network element values and the undamped natural frequency, the L/R time constant and the RC time constant (as well as others). For example, if you are given the values of R, L and C for a parallel-resonant network, without any other information you can compute both the undamped natural frequency, from Equation (16) above, and the network Q, with Equation

((24). No additional information is needed. Specifically you do not need to know at what frequency you are operating the network.

Now, does it make sense that the equation for Q should be different for a parallel-resonant network as compared to a series-resonant network? Q is a measure of the losses of a resonant network — the higher the Q the lower the network losses. You might say that Q is a measure of the “loss-lessness” of the network. So, it seems that the more resistance that you have in a network the higher the loss and the lower the Q must be, but that is not quite right. Consider the parallel network of Figure 1 for a moment. Suppose that we start out with some value of resistance and some specific drive voltage. We can easily compute the power dissipated in the network (Equation (8) above) since we know the resistance value and the voltage across it. Now suppose that we keep the drive voltage the same but we make the resistor value *larger*. The power dissipated will go down since $P = E^2/R$. Since Q is a measure of loss-lessness, we would expect Q to increase since the loss decreased. And from Equation (25) we see that this is true. As we make the resistance value higher, the Q_p becomes higher, and at an infinite value of resistance, the value of Q_p is also infinite. So for the parallel network, *higher* resistance means higher Q_p.

Now consider the series network if Figure 2. Suppose that we have a specific drive current and a specific value of resistance. Again we can compute the power dissipated in the network since we know the resistance and the current through it ($P = I^2 R$). Now if we wish to reduce the power dissipation, we must *reduce* the resistance value. So, as we reduce the resistance value of the series network, the value of Q_s will increase. This is shown by Equation (33). If the resistance value is made zero, the value of Q_s will become infinite.

So, the computation of the value of Q should be different for the parallel and series networks. This is an important point to remember.

Now let's look briefly how we may misuse the circuit parameter Q. Look back at Equation (33). Look for example at the expression of Q in terms of the inductance. It appears that in this form, the value of Q is independent of the capacitance. This is not really true because the term ω_0 is a function of the capacitance, but suppose that we ignore this fact too. Now suppose that we change the very specific ω_0 term to a general ω term. We cannot properly do this, but suppose that we do it anyway. Now rewriting Equation (33) in terms of our improper use of ω we get the following expression. I did not give this an equation number since I want to make sure that this is not mistaken for a valid equation for Q!

$$Q_p = \frac{\omega L}{R}$$ <u>IMPROPER EQUATION</u>!

Unfortunately, this expression probably looks all too familiar. This expression looks as though if we were given an inductor with its resistance value, inductance value and some frequency, we could compute the Q value of this inductor. For example, if we have an inductor, say 1μH, that has a resistance of say 1 ohm that we use as a series power-supply inductance to reduce power-supply noise in a 10.7MHz IF amplifier, it appears that we can calculate the Q of this inductor from the improper equation above as 63 from $\omega L/R$. This is not correct. From Equations (24) and (32) we see that we must have some specific network capacitance to compute Q. And even more confusing, if we were to try to apply those equations, which would we use, the parallel case or the series case?

When we look at inductor specifications, one of the parameters that is given is the Q of the inductor, usually at some frequency (hopefully). Just what does that mean? Well, whatever the manufacturer had in mind when it made that specification. It could be the Q at the self resonance of the inductor, the self-resonant natural frequency, but recall that the natural frequency is not the same as the undamped natural frequency. Or it could be that if the inductor is resonated with some capacitor at the specified frequency, this is the Q that will result. This is most likely what the inductor Q specification typically means. But, is that the natural frequency or is it the undamped natural frequency? And is it parallel resonant or series resonant? If the Q is high, (>~10), the natural frequency and the undamped natural frequency will be very close together so it is not too important which is used in a practical sense. But it is important if you wish to understand the basic concept of Q. As for series or parallel resonance, either may be used depending what you are interested in. For example, if a series-resonant network is assumed, the specified Q value, and a corresponding capacitance computed from the specified frequency and specified inductance, may be used to compute the *equivalent series resistance* of the inductor from Equation (32). And, if a parallel network is assumed, the specified Q value may be used to compute the *equivalent parallel resistance* of the inductor using Equation (24). Both of these resistances are perfectly valid, and indeed given either, the other may be computed. They are simply different models of the inductor — ideal inductor in series with a series resistance, or ideal inductor in parallel with a shunt resistance.

The most important point to understand from this paper is that the parameter Q is a true network parameter as shown by Equations (24) and (32), and that Q is a function of ***only*** the network parameters R, L, and C of a second-order, resonant network. And equally as important as shown by Equations (24) and (32), Q is ***not*** a function of the operating frequency of the network just as the values of the network resistance, capacitance or inductance, or the L/R or RC time constants, or the undamped natural frequency $1/\sqrt{LC}$ are not a function of the operating frequency. This is a very important concept to always remember when applying the parameter Q.

If we were to “charge” an R-L- C network, for example by putting a charge in the capacitor, and allow it to “ring down,” what we will see, if the “Q” is high enough, is an exponentially decaying sinusoidal signal. For example, in a parallel R-L-C network, if we charge the capacitor to some voltage and then let the network go, we will see the voltage across the network oscillate sinusoidally and decay exponentially in amplitude with time. The exact equation of the network voltage is given by Equation (34).

$$V(t) = V_0 e^{-\frac{L}{2R}t} Cos(\omega_n t) \tag{34}$$

were $\omega_n = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = \sqrt{{\omega_0}^2 - \left(\frac{R}{2L}\right)^2}$

The frequency ω_n is the free ringing frequency, or *natural frequency*, of the network. It is important to notice that it is not equal to the undamped natural frequency ω_0. The effect of the resistance in the circuit is to lower the free ringing frequency below the natural frequency. If we substitute Equation (16) into Equation (34) and we solve Equation (23) for L and also substitute into Equation (34), the dynamical response as a function of Q may be found.

$$V(t) = V_0 e^{-\left(\frac{\omega_0}{2Q}\right)t} Cos(\omega_n t) \tag{35}$$

were $\omega_n = \omega_0 \sqrt{1 - \frac{1}{4Q^2}^{\,2}}$

Also, it can be shown that Equation (35) applies to both parallel and series networks.

Now for a practical example. Suppose that we have a nominal 2.5 meter monopole antenna which we wish to match to a 50-ohm transmitter at 5MHz. If we do a good job on the ground plane, at the feed point this antenna will look like about a one-ohm radiation resistance in series with about a 26pf capacitance. Let's match this antenna with a simple series inductance and a shunt capacitance. I call these two elements the compensating inductance and the matching capacitance. Let the transmitter frequency be f_T, or ω_T in angular form. The equivalent circuit is shown in Figure 3. In Figure 3 I have shown the compensating inductance L_c as two inductors in series. One of these, L_r, is the inductance required to resonate the antenna capacitance at the operating frequency to make the feed point look resistive and exactly equal to the antenna radiation resistance. The second inductor, L_m, is the inductance required in the L-C matching network to match the radiation resistance to the transmitter 50-ohm impedance. Of course in actual practice, these two inductors would simply be a single inductor, L_c, equal to the sum of the two series inductors. Separating the inductor into two individual inductors makes it easier to see what is going on. I have included just enough of the math here so that you can see what is happening.

First, we must compute the value of L_r required to resonate the antenna capacitance at the desired frequency. Note that L_r, R_r and C_a from a series R-L-C network. At the undamped natural frequency, the impedance of a series R-L-C network is real and equal to the value of the series resistance. The undamped natural frequency is given by Equation (16) above. We know the frequency at which we want to match, and we know the antenna capacitance. We can then compute the inductance L_r that we must use to resonate the antenna capacitance at our operating frequency.

$$L_r = \frac{1}{\omega_T{}^2 C_a} = \frac{1}{\left(2\pi \cdot 5\,MHz\right)^2 \cdot 26\,pf} = 38.97\,\mu H \tag{36}$$

Note again that this is not the inductance we need to match the antenna to our transmitter. This is the inductance we need to resonate the antenna capacitance to make the antenna impedance real. If we simply place about 39μH in series with the antenna at the feed point, at 5MHz the impedance seen looking into the antenna with this series inductance will be simply the antenna radiation resistance, or about one ohm (ignoring the losses in the 39μH inductor).

Now we need to match this one-ohm resistance to the 50-ohm transmitter impedance. The equivalent circuit of the matching network with the antenna capacitance resonated is shown in Figure 4. The impedance Z_{in} seen looking into the network may be written by inspection (yes it really can).

$$Z_{in} = \frac{1}{j\omega_T C_m} // \left(L_m + R_a\right)$$ (the notation "//" means in parallel with)

$$= \frac{R_a + j\omega_T L_m}{\left(1 - \omega_T{}^2 L_m C_m\right) + j\omega_T C_m} \tag{37}$$

If the network impedance is real at some frequency, which is what we are trying to achieve, then the imaginary part of Equation (37) must be zero at that frequency. Solving for the imaginary part and setting it equal to zero, we can solve for the matching inductance L_m and the compensating capacitance C_m.

$$L_m{}^2 = \frac{R_r\left(R_{in} - R_r\right)}{\omega_T{}^2} \tag{38}$$

$$C_m{}^2 = \frac{\left(R_{in} - R_r\right)}{\omega_T{}^2 R_{in}{}^2 R_r} \tag{39}$$

We can also solve for the Q of the matching network by substituting the values for L_m and C_m from Equations (38) and (39) into the expression for Q. Let this Q for the matching network value be Q_m.

$$Q_m = \frac{1}{R_r}\sqrt{L_m / C_m} = \frac{1}{R_r}\sqrt{R_{1n} R_r} = \sqrt{R_{in} / R_r} \tag{40}$$

Note in Equations (38) and (39) that the matching inductance and matching capacitance are functions only of the antenna radiation resistance, the desired input resistance to the compensated network and the operating frequency. This operating frequency is not the undamped natural frequency. Also, note from Equation (40) that the value of Q_m is simply the square root of the ratio of the source resistance to the antenna resistance. The physical significance of Q_m is that it is square of the resistive matching ratio of the source resistance to the load resistance. For our 50-ohm source matched to the 1-ohm antenna resistance, Q_m is 7.07.

$$Q_m = \sqrt{\frac{50Ohms}{1Ohm}} = 7.07$$

Computing the values of matching inductance and matching capacitance,

$$L_m = \left[\frac{1Ohm(50Ohms - 1Ohm)}{(2\pi \cdot 5MHz)^2}\right]^{1/2} = 0.223\mu H \tag{41}$$

$$L_c = L_m + L_r = 39.193\mu H \tag{42}$$

$$C_m = \left[\frac{(50Ohms - 1Ohm)}{(2\pi \cdot 5MHz)^2 (50Ohms)^2 1Ohm}\right]^{1/2} = 4{,}456pf \tag{43}$$

So, theoretically, if we place a 39.193μH inductor (38.97μH +0.223μH) in series with the antenna at the feed point and then place a 4,456pf capacitor in shunt, at the 5MHz transmitter frequency we should see a 50-ohm resistance looking into the compensated antenna. I simulated this using Electronics Workbench and the results are shown in Figure 5. As expected, at 5MHz the magnitude of the input impedance is almost exactly 50 ohms and the phase angle is almost exactly zero so the impedance is real. The antenna is indeed almost perfectly matched to the 50-ohm transmitter at 5MHz.

Another interesting parameter of an antenna is the current in the antenna, I_a, and the voltage developed at the antenna feed point, V_a. The source voltage applied to the input to the matching network is simply the transmitter output voltage, V_s. The antenna current may be simply written by inspection of Figure 4 as the source voltage divided by the impedance seen looking into the matching inductance. The voltage appearing at the antenna feed point, that is across the series combination of the antenna resistance and the antenna capacitance, may be written by inspection of Figure 4 using several approaches. First we will compute the antenna current. At the operating frequency, the resonating inductance L_r exactly resonates with the antenna capacitance C_a. Therefore, the impedance seen by the source looking into the matching inductance L_m is simply the matching inductance in series with the antenna resistance. The current driven into this branch of the circuit, which is conveniently the antenna current, is simply the source voltage applied at the input to the matching network divided by the sum of the antenna resistance and the reactance of the matching-inductance.

$$I_a = \frac{V_s}{R_a + j\omega_T L_m} \tag{44}$$

$$\left|I_a\right| = \frac{V_s}{\sqrt{R_a^{\,2} + \omega_T^{\,2} L_m^{\,2}}} \tag{45}$$

We will come back to Equation (45) a little later.

One straight-forward means to compute the antenna voltage is to recognize that the matching inductance, the resonating inductance, the antenna resistance and the antenna capacitance form a simple voltage divider. The voltage developed across the antenna at the antenna feed point may then be simply computed by the voltage division between the matching and resonating inductances and the antenna impedance (antenna resistance plus antenna capacitance).

$$V_a = V_s \frac{R_a + \dfrac{1}{j\omega_T C_a}}{R_a + \dfrac{1}{j\omega_T C_a} + j\omega_T L_m + j\omega_T L_r}$$

$$V_a = V_s \frac{1 + j\omega_T R_a C_a}{-\omega_T^{\,2} L_m C_a - \omega_T^{\,2} L_r C_a + j\omega_T R_a C_a + 1} \tag{46}$$

At the operating frequency ω_T the antenna capacitance is exactly resonant with the resonating inductor. Therefore,

$$\omega_T = \frac{1}{\sqrt{L_r C_a}}$$

Substituting this value for ω_T into Equation (46) and simplifying,

$$V_a = V_s \frac{1 + jR_a\sqrt{\frac{C_a}{L_r}}}{-\frac{L_m}{L_r} + jR_a\sqrt{\frac{C_a}{L_r}}} \tag{47}$$

We of course immediately recognize that the term $R_a\sqrt{C_a/L_r}$ is the reciprocal of a Q term. This term is the Q of the resonant network comprising the resonating inductor, the antenna resistance and the antenna capacitance. Let this Q term be defined as Q_r.

$$Q_r = \frac{1}{R_a}\sqrt{\frac{L_r}{C_a}} \tag{48}$$

Substituting this value of Q_r into Equation (47) and simplifying,

$$V_a = -V_s \frac{\left(\frac{L_m}{L_r}Q_r^{\ 2} + 1\right) + jQ_r\left(\frac{L_m}{L_r} - 1\right)}{\left(\frac{L_m}{L_r}\right)^2 Q_r^{\ 2} + 1} \tag{49}$$

$$|V_a| = V_s \frac{\sqrt{\left(\frac{L_m}{L_r}Q_r^{\ 2} + 1\right)^2 + Q_r^{\ 2}\left(\frac{L_m}{L_r} - 1\right)^2}}{\left(\frac{L_m}{L_r}\right)^2 Q_r^{\ 2} + 1} \tag{50}$$

The negative sign in Equation (49) simply shows that the antenna voltage will be almost 180° out of phase with respect to the source voltage if the value of Q_r is high enough. In fact, if Q_r is relatively high and the ratio of L_m to L_r relatively low, the magnitude of the antenna voltage will be approximately equal to the source voltage multiplied by the ratio of L_r divided by L_m.

$$|V_a| \cong \left(\frac{L_r}{L_m}\right) V_s \tag{51}$$

$$\text{for } Q_r >> 1 \text{ and } {}^{L_m}\!/_{L_r} << 1$$

The physical meaning of Equations (50) and (51) is that these give the voltage multiplication factor from the input to the matching network to the voltage at the antenna feed point (the "Tesla-Coil" effect). For example, consider that we wish to radiate 100W at 5mhz from our antenna. The transmitter source voltage at the input to the matching network will be 70.7V rms(100W into 50 ohms). Then applying Equation (50), the voltage that would appear at the antenna feed point would be approximately 12.2kV rms (173 times the transmitter voltage). If we use the approximate expression of Equation (51) we get 12.4kV which is a very good approximation. This voltage multiplication is why one must use high-voltage components and good high-voltage construction techniques in antenna-matching networks even where only modest powers are to be radiated. In a "real" antenna system, the total antenna resistance will be much higher than one ohm, in other words the Q values will be much lower, (Q values of about 100 to perhaps 200 are about the best that we can expect without very elegant techniques), so the voltage multiplication will be substantially lower. By the way, this voltage-multiplication property of a series-resonant second-order network is a great science project.

So, the antenna voltage is very much higher than the transmitter voltage. Then what about the antenna current? If we substitute the value for ω_T into Equation (45) and simplify, the magnitude of the antenna current is then given by Equation (52).

$$\left|I_a\right| = \frac{V_s}{\sqrt{R_a^{\;2} + \omega_T^{\;2} L_m^{\;2}}} = \frac{V_s}{\sqrt{R_a^{\;2} + \frac{L_m^{\;2}}{L_r C_a}}}$$

$$\left|I_a\right| = \frac{V_s}{R_a \sqrt{1 + \left(\frac{L_m}{L_r}\right) Q_Z^{\;2}}} \tag{52}$$

$$\text{where } Q_Z = \frac{1}{R_a}\sqrt{\frac{L_m}{C_a}}$$

The transmitter current it simply the transmitter voltage divided by 50 ohms since the matched impedance is 50 ohms, or about 1.4A rms for a 100W power level ($1.4A^2$ x 50 ohms = 100W). From Equation (52), we find that the magnitude of the antenna current is 10A rms, or about 7 times the transmitter current. And indeed, this 10A antenna current flowing through the 1-ohm antenna resistance results in a 100W radiated power. Convenient how that works. But where did all this current come from if the transmitter current is only 1.4A? Well, it is provided by the matching capacitance C_m. This capacitance stores energy and delivers it into the network at just the right time, that is with just the right phase, to provide the added current required in the antenna to radiate the desired power. So, the antenna matching network provides both a voltage multiplication and a current multiplication for the specific antenna case examined here.

It is useful to note in this antenna example that a Q_m of only about 7 is required to match the resistive part of the load to the source impedance, but a Q_r of greater than 1,700 is needed to achieve the necessary voltage multiplication. This antenna will be very difficult to keep in tune for minimum VSWR and maximum radiated power due to this high value of Q_r. This is one of the disadvantages of an antenna that is very short compared to a quarter wavelength.

One final useful application of Q is related to bandwidth of a tuned circuit. Generally, we compute the bandwidth as the center frequency divided by the Q. But, just where does this come from? And is it a valid use of the parameter Q, or is it just one of those concepts that has just sort of come into being? As I will show below, this is a valid use of Q provided we are talking about a simple second-order tuned circuit.

Bandwidth is defined as the frequency difference between the upper half-power frequency and the lower half-power frequency, also called the half-power points. A half-power frequency is simply a frequency at which the power delivered to a load is one half the power delivered at the frequency of maximum power delivery. Since a reduction in power by a factor of two is a power change of -3dB, the upper and lower half-power frequencies are also called the upper and lower -3dB frequencies, or simply the 3dB points, and the bandwidth the 3dB bandwidth.

Figure 6 shows a simple parallel R-L-C tuned circuit driven by a current source. Here the math is simplifier if we use a current source excitation. The resistance in this resonant network is the load. This network meets the requirements of a simple driven harmonic oscillator, so the parameter Q may be properly applied to this network. Since this is a parallel resonant circuit, its Q_p is given by Equation (24) above.

When this network is excited at its undamped natural frequency, its impedance is real and equal to the load resistance. Therefore, at this frequency, if the peak value of the exciting current source is I_0, the power delivered to the load is given by Equation (53).

$$P_L @ \omega_0 = \left(I_0 \big/ \sqrt{2} \right)^2 R_L = \frac{1}{2} \frac{I_0^{\ 2}}{R_L} \tag{53}$$

For this network, the undamped natural frequency is the frequency at which the highest power is delivered to the load resistance. Now we must compute the upper and lower half-power frequencies. Since power dissipated in the resistor is proportional to the square of the voltage across the resistor, when the voltage is reduced by $\sqrt{2}$, the power is reduced by 2, or to one half the power. Then, at the half-power frequency, the voltage developed across the load resistance is equal to the voltage developed at the point of maximum power divided by $\sqrt{2}$. This voltage reduction by $\sqrt{2}$ will occur when the magnitude of the impedance of the network is reduced by $\sqrt{2}$ below its value at the undamped natural frequency. Since this is a parallel network, it is more convenient to work with admittances of the elements. The admittance of the network may be written by inspection.

$$Y_N = \frac{1}{j\omega_T L} + j\omega_T C + \frac{1}{R_L}$$

$$= \frac{\omega_T L - j\left(R_L - \omega_T R_L LC\right)}{\omega_T R_L L} \tag{54}$$

Indeed, when $\omega_T = \omega_0$, Equation (54) reduces to $1/R_L$ — remember Equation (54) is the admittance of the network which is one over the impedance. So, at the half-power frequencies, the magnitude of the admittance is $\sqrt{2}/R_L$. Since the magnitude of the admittance involves some squares and square roots, it is a little more convenient to use the square of the admittance. So, at the half-power frequencies,

$$\left|Y_N\right|^2 \equiv \frac{2}{R_L} \qquad \text{@ half-power points}$$

The magnitude of the admittance is simply the square root of the sum of the squares of the real part and the imaginary part of Equation (54), and the magnitude squared is simply the square of that square root. This is why using the squares is a bit more convenient — eliminates having to carry a bunch of square-roots around. Then,

$$\left|Y_N\right|^2 = \frac{\omega_T{}^2 L^2 + \left(R_L - \omega_T{}^2 R_L C\right)}{\omega_T{}^2 R_L{}^2 C^2} \equiv \frac{2}{R_L{}^2}$$

$$\left(R_L - \omega_T{}^2 R_L LC\right) - \omega_T{}^2 L^2 = 0$$

$$\left[\left(R_L - \omega_T{}^2 R_L LC\right) - \omega_T L\right] \cdot \left[\left(R_L - \omega_T{}^2 R_L LC\right) + \omega_T L\right] = 0$$

$$\left(R_L LC\omega_T{}^2 + L\omega_T - R_L\right) \cdot \left(R_L LC\omega_T{}^2 - L\omega_T - R_L\right) = 0 \tag{55}$$

Now Equation (55) has four roots (four values of ω_T that satisfy the equation): two at negative frequency and two at positive frequency. The two positive-frequency roots are the ones we need since the negative roots do not have physical meaning to our problem here. Applying the quadratic equation, combining terms and substituting $1/LC \equiv \omega_0{}^2$,

$$\omega_T(1,2,3,4) = \frac{\pm L \pm \dfrac{1}{\omega_0}\sqrt{\dfrac{L}{C} + 4R_L{}^2}}{2R_L LC} \tag{56}$$

The magnitude of the second term in the numerator is always larger than L, so the two positive roots are the positive second term plus and minus the first term.

$$\omega_T(1) = \frac{+L + \dfrac{1}{\omega_0}\sqrt{\dfrac{L}{C} + 4R_L{}^2}}{2R_L LC} \Rightarrow \text{Upper Half-Power Frequency} \tag{57}$$

$$\omega_T(2) = \frac{-L + \dfrac{1}{\omega_0}\sqrt{\dfrac{L}{C} + 4R_L{}^2}}{2R_L LC} \Rightarrow \text{Lower Half-Power Frequency} \tag{58}$$

Now the bandwidth is defined simply as the upper half-power frequency minus the lower half-power frequency. The notation BW_a is used to denote that the bandwidth is in terms of angular frequency (radians). Since $\omega = 2\pi f$, the bandwidth BW_f in terms of frequency f is simply $BW_a / 2\pi$.

$$BW_a(-3dB) = \frac{+L + \frac{1}{\omega_0}\sqrt{\frac{L}{C} + 4R_L{}^2}}{2R_L LC} - \frac{-L + \frac{1}{\omega_0}\sqrt{\frac{L}{C} + 4R_L{}^2}}{2R_L LC}$$

$$= \frac{+L + \frac{1}{\omega_0}\sqrt{\frac{L}{C} + 4R_L{}^2} + L - \frac{1}{\omega_0}\sqrt{\frac{L}{C} + 4R_L{}^2}}{2R_L LC}$$

$$BW_a(-3dB) = \frac{1}{R_L C} \tag{59}$$

This result in Equation (59) is useful since it gives us the angular bandwidth (radians/second) in terms of the network resistance and capacitance. But, what we are trying to find is bandwidth as a function of Q. We are getting close. If we go back to Equation (16), solve for C and substitute in Equation (59), and fiddle with the terms a little we get,

$$BW_a(-3dB) = \frac{\omega_0{}^2 L}{R_L} = \omega_0\left(\frac{\omega_0 L}{R_L}\right) = \frac{\omega_0}{\left(\frac{R_L}{\omega_0 L}\right)} \tag{60}$$

And if we look back at Equation (25), we see that the denominator term in brackets is Q_p for a parallel R-L-C network (convenient how that too works out isn't it?). Then,

$$BW_a(-3dB) = \frac{\omega_0}{Q} \tag{61}$$

If we divide both sides of Equation (61) by 2π, the left side of the equation will become BW_f and the right side ${f_0}/{Q}$.

$$\frac{BW_a(-3dB)}{2\pi} = \frac{\left(\frac{\omega_0}{2\pi}\right)}{Q}$$

$$BW_f = \frac{f_0}{Q} \tag{62}$$

And,

$$Q = \frac{f_0}{BW_f} \tag{63}$$

So, there we have it. The half-power bandwidth is indeed given by the undamped natural frequency divided by Q. Conversely, Q is given by the undamped natural frequency divided by the bandwidth. This also holds for a series R-L-C network (the proof of this is left as an exercise to the reader!).

Suppose that we have a double-tuned IF stage. This stage provides a very nice response with very well-defined half-power points. So, it seems that we should be able to compute the "Q value" of this stage by simply dividing the center frequency by the bandwidth. Well, no you cannot really do that. Recall that the fundamental definition of Q is only for a simple, second-order harmonic oscillator. The double-tuned IF is not second-order. It is some higher order since each tuned circuit is second order, and they are coupled as well. Q is not defined for this type of circuit. However, since it is well known that Q for a simple second-order network is given by the center frequency over the bandwidth, it is a simple jump to set aside the fundamental definition of Q and use this center-frequency/bandwidth ratio to compute Q values for virtually any type of circuit that provides a response that looks like a tuned-circuit response. This is an improper use of the Q parameter. We sort of know what Q means in this case, but if you try to compute circuit parameters using such a Q value, the results will not be at all as expected. For example, if you are designing a double-tuned 10.7MHz IF stage with a 300kHz bandwidth, you may compute the required "Q" of network as 35.7. But if you try to compute the circuit component values of the double-tuned network from this Q value using the results derived above, the response will not be as expected since the equations derived above do not apply in this case of a double-tuned network.

We could go on almost indefinitely looking at various applications of this parameter Q, both proper and improper. But I believe that the basics have been covered reasonably well above. So summarizing very briefly, we have seen that Q is a true network parameter totally defined by the component values of the network, and specifically it is not related to the actual frequency at which we are operating the network. The Q parameter applies only to simple second-order networks. It applies to both series and parallel networks, but the value of Q in terms of the network component values are different for the series and parallel case. And finally, we find Q used in many ways in the art, some proper and some improper, and we must be cautious when using this parameter to be sure we use it appropriately, and we must be careful that we are sure we understand what is intended when we see it used. If you always keep in mind Equation (24), $Q_p = R\sqrt{C/L}$, and Equation (32), $Q_s = \frac{1}{R}\sqrt{L/C}$, it should help eliminate confusion.